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NATURAL CONVECTION IN SATURATED POROUS LAYERS WITH INTERNAL HEAT SOURCES

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Abstract—The problem of two-dimensional convection in an internally heated saturated porous layer has been investigated numerically by means of finite differences. The porous layer is superposed by a layer of fluid and arranged on an impermeable and adiabatic base. Limiting to small permeabilities a constant pressure is assumed at the interface between the saturated porous medium and the superposed liquid layer, and the temperature is fixed there by an empirical heat flux correlation for fully turbulent convective flow in the liquid layer. Starting from initial temperature distributions, steady-state convection was found for given Rayleigh numbers up to a hundred times the critical value. Nusselt and Fourier numbers are presented in the form of power laws of the Rayleigh number.

NOMENCLATURE

B	width of the porous layer
c_p, c_L	specific heat of the porous bed and the liquid, respectively
c^*	fraction of heat capacity, $(\rho c)_L/(\rho c)_p$
Fo	Fourier number (as dimensionless transient period of the convection within the porous bed)
g	acceleration of gravity
H_1, H_2	depth of the porous layer and of the overlying liquid layer, respectively
K	permeability
Nu	mean Nusselt number, $QH_1^2/\lambda_p\langle\theta_p-\theta_L\rangle$
Nu_L	local Nusselt number of the overlying liquid layer, $q_1H_2/\lambda_L(\theta_1-\theta_L)$; where q_1 is the local heat flux through the interface
p	pressure, dimensionless scaled with $\nu\kappa_p\rho_L/(Kc^*)$
Pr	Prandtl number, ν/κ_L
Q	heat generation rate per unit volume of the particle bed
Ra_I	internal Rayleigh number, $\alpha gQKc^*H_1^3/(\lambda_p\kappa_p\nu)$
Ra_L	local Rayleigh number of the overlying liquid layer, $\alpha gH_2^3(\theta_1-\theta_L)/(\nu\kappa_L)$
Ra_δ	Rayleigh number of the boundary layer, $\alpha gK\delta c^*\langle\theta_p-\theta_1\rangle/(\nu\kappa_p)$
t	time, dimensionless scaled with H_1^2/κ_p
T	dimensionless temperature difference to the upper surface of the liquid layer, scaled with QH_1^2/λ_p
T_p, T_1	dimensionless temperature difference of the bottom of the porous layer and of the bed–liquid layer interface, respectively
u	horizontal velocity, dimensionless scaled with $\kappa_p/(H_1c^*)$
v	vertical velocity, dimensionless scaled with $\kappa_p/(H_1c^*)$
v_m	mean velocity through the interface, defined by equation (22)
v_{max}	maximum velocity in the porous bed, scaled as v
x	horizontal coordinate, dimensionless scaled with H_1
$\Delta x, \Delta y$	width of finite differences, scaled as x or y
y	dimensionless distance from the bottom of the porous layer, scaled with H_1.

Greek symbols

α	thermal expansion coefficient
δ	thickness of the boundary layer at the interface
θ	temperature
$\theta_p, \theta_1, \theta_L$	temperatures of the bottom of the porous bed, of the bed–liquid layer interface and of the upper surface of the liquid layer, respectively
κ_p, κ_L	thermal diffusivity of the porous medium and the liquid, respectively
λ_p, λ_L	thermal conductivity of the porous medium and the liquid, respectively
ν	kinematic viscosity
ρ_p, ρ_L	mean density of the porous medium and of the liquid, respectively.

Subscripts

p	porous bed
L	liquid.

Brackets

$\langle\ \rangle$	horizontally averaged values.

1. INTRODUCTION

IN THE risk assessment of nuclear power plants, the possibility and the consequences of a melt down of the reactor core are usually considered. During the course of such an accident, molten fuel and coolant may interact. Violent thermal reactions can disperse the molten fuel into fine particles. These small particles quickly solidify in the coolant and settle on internal structures of the reactor pressure vessel forming a saturated porous particle bed. The question arises, under what conditions the nuclear decay heat can be removed from the particle bed to the ambient coolant by natural convection. The posed problem applies for light water reactors as well as for sodium-cooled fast breeder reactors. As far as the application in this work is concerned, reference is made only to the latter type of reactor.

The onset of convection in an internally heated saturated porous layer has been studied by Gasser and Kazimi [1] and Turland [2]. Turland calculated the critical Rayleigh number for a porous layer which is superposed by a layer of coolant. The effect of a superposed coolant layer was taken into account by introducing an integral heat transfer coefficient for the liquid layer and by assuming a constant pressure on the upper side of the porous layer. The latter condition means that the coolant can be freely exchanged between the porous and the liquid layer.

Such a model must have limited validity, since in reality these conditions are not readily definable at the interface between the two layers, depending on the particle size and the respective heights of the layers. Conditions at the interface in more general form have been used by Nield [3] and discussed in a related context by Beavers and Joseph [4].

In the present study, the investigations of Turland were extended for the double layer to seek an answer to the question of convective heat transfer at high supercritical Rayleigh numbers. Moreover, the validity of the simplified thermal and kinematic conditions at the interface which have been used for the calculation of temperature and velocities in the porous layer is supported by analytical considerations and by comparison with experimental results.

The numerical calculations were performed by means of finite differences. A similar procedure was previously used by Ribando and Torrance [5] for predicting natural convection in a saturated porous layer heated from below under various boundary conditions.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Let us consider a horizontal, porous layer of thickness H_1 and permeability K. The porous medium is saturated with a fluid of density ρ_L and viscosity ν and superposed by a layer of the same fluid. The fluid layer has a thickness H_2. The upper boundary of the fluid layer is kept at a constant temperature θ_L, whereas the lower side of the porous layer is thermally insulated, i.e. adiabatic. The porous medium is heated by homogeneously distributed heat sources of strength Q. Thermal conductivities λ and specific heats c of both fluid and porous matrix were, as well as the thermal expansion coefficient α of the fluid, taken to be constant. Generally the subscripts L and p denote the properties of the liquid and the porous medium composed by liquid and particles, respectively. In the absence of fluid motion, the temperature varies parabolically across the porous medium and linearly across the liquid layer. The fluid motion is assumed to be a two-dimensional (2-D) roll convection. This simplification is justified for most engineering applications, where predictions of integral heat transfer rates are of primary interest.

The governing equations for natural convection in a porous layer with internal heat sources can be written with the Boussinesq approximation and the Darcy flow, with negligible inertia terms in the following dimensionless form [6]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial p}{\partial x}+u=0, \tag{2}$$

$$\frac{\partial p}{\partial y}+v=Ra_I T \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]T+1. \tag{4}$$

In these equations, length, time, velocity, pressure, and temperature are scaled by H_1, H_1^2/κ_p, κ_p/H_1c^*, $(\nu\kappa_p\rho_L)/(Kc^*)$, $(QH_1^2)/\lambda_p$, respectively, where $c^*=(\rho c)_L/(\rho c)_p$. Moreover, θ_L is chosen as the reference temperature. Thus $T_L=0$. The dimensionless quantity Ra_I, the so-called internal Rayleigh number, is given by the relation

$$Ra_I=\frac{\alpha g Q K c^* H_1^3}{\lambda_p\kappa_p\nu}, \tag{5}$$

where g is the acceleration due to gravity.

Introducing a Cartesian coordinate system as indicated in Fig. 1 we obtain for the dimensionless boundary conditions at the lower side of the porous layer

$$v=0,\quad \frac{\partial T}{\partial y}=0\quad \text{at } y=0. \tag{6}$$

If we limit our considerations to porous media of small dimensionless permeability K/H_1^2 of the order of

$$K/H_1^2 \lesssim 10^{-7},$$

a constant pressure can be assumed at the interface (see Appendix)

$$p=\text{const.}\quad \text{at } y=1. \tag{7}$$

This condition implies that the (turbulent) flow in the superposed fluid is hydraulically decoupled from the seepage flow in the porous layer.

Assuming furthermore that the thickness H_2 of the superposed fluid layer is of the same order or much larger than the thickness H_1 of the porous layer, the

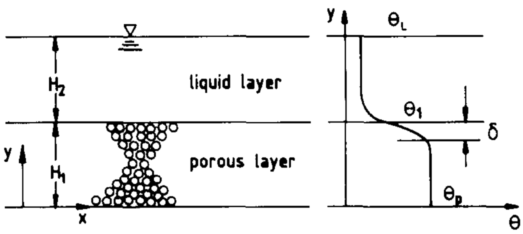


FIG. 1. Schematic sketch of the two-layer models.

dimensionless temperature T_1 at the interface can be linked to the temperature gradient by an empirical heat flux correlation for the turbulent Bénard convection in the fluid layer. Balancing the heat flow rate at the interface and introducing a Nusselt number for the fluid layer we obtain the following condition for the temperature

$$\langle T\rangle\, Nu_L(\langle T\rangle) = -\frac{\lambda_p}{\lambda_L}\frac{H_2}{H_1}\left\langle\frac{\partial T}{\partial y}\right\rangle \quad \text{at } y=1, \tag{8}$$

where a horizontal average of the temperature is indicated by angular bracket, $\langle\ \rangle$. For numerical calculations this condition is finally simplified by dropping the horizontal averaging for the heat flux yielding

$$T\, Nu_L(T) = -\frac{\lambda_p}{\lambda_L}\frac{H_2}{H_1}\frac{\partial T}{\partial y} \quad \text{at } y=1. \tag{8a}$$

The above procedure can be argued physically, if we assume that the thickness of the liquid layer is much larger than that of the porous layer, i.e. $H_2/H_1 \gg 1$, and that the convective flow in the liquid layer is fully turbulent. These assumptions are often valid for problems mentioned in the Introduction. For other problems, the range of this simplified condition would have to be scrutinized by comparison with experimental results or by analytical considerations such as those presented in Sections 5 and 6, respectively.

For further treatment of the problem, turbulent heat transfer will be introduced in the fluid layer. According to the experimental results of O'Toole and Silveston [7] the following correlation holds for turbulent free convection in fluid layers heated from below

$$Nu_L = 0.104 Ra_L^{0.305}\, Pr^{0.084}, \quad 10^5 \leqslant Ra_L \leqslant 10^9, \tag{9}$$

where the Rayleigh number Ra_L and the Prandtl number Pr of the fluid layer are, respectively, defined by

$$Ra_L = \frac{\alpha g(\theta_1-\theta_L)H_2^3}{\nu\kappa_L}, \quad Pr = \frac{\nu}{\kappa_L}. \tag{10}$$

The 'internal' Rayleigh number Ra_I [equation (5)], the dimensionless temperature T_1 and the Rayleigh number for the fluid layer are related as follows

$$Ra_L = Ra_I T_1 \frac{H_1^2}{K}\frac{H_2^3}{H_1^3}\frac{\lambda_p}{\lambda_L}. \tag{11}$$

The governing differential equations, equations (1)–(4), together with the boundary conditions, equations (6)–(8), form a complete set of relations for determining the velocity, pressure and temperature distributions in the porous layer.

3. NUMERICAL PROCEDURE

Finite differences are used to approximate the partial differential equations, equations (1)–(4). Starting from initial data of the temperature field in a first step, the pressure is determined from the Poisson equation of the form

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)p = Ra_I\frac{\partial T}{\partial y}. \tag{12}$$

Point central differences are used here for all space derivatives. The velocity components u and v are calculated from equations (2) and (3). An explicit scheme with forward time differencing is used to approximate the time derivative in the energy equation, equation (4). The spacial derivatives in this equation are replaced by so-called 'exponential differences' [8] to assure stability at high mesh Péclet numbers. According to this scheme the sum of the convective and diffusive terms in the energy equation containing partial derivatives with respect to x is approximated by the relation

$$\left[u\frac{\partial T}{\partial x}-\frac{\partial^2 T}{\partial x^2}\right]_i \approx u_i\frac{T_{i+1}-T_{i-1}}{2\Delta x} - \frac{u_i}{2}\coth\left[\frac{u_i}{2}\right]\frac{T_{i+1}+T_{i-1}-2T_i}{(\Delta x)^2}. \tag{13}$$

A corresponding relation is used for similar terms in equation (4) with partial derivatives with respect to y.

A somewhat different approximation is used at a boundary, where the temperature is not fixed. There the boundary temperature is evaluated from the solution of the differential equation

$$u\frac{\partial T}{\partial x}+\frac{\partial^2 T}{\partial x^2} = \text{const.},$$

which is

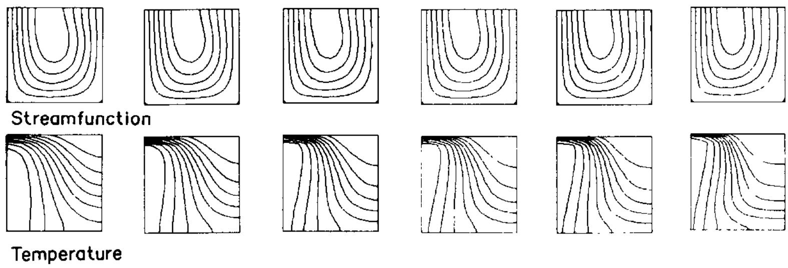
$$T = k_0 + k_1 x + \frac{k_2}{u}\,e^{ux},$$

if u is assumed to be constant within a given mesh size. Therein the constants k_0, k_1, k_2 have been determined from two mesh points next to the boundary and by complying with a boundary condition of the general form

$$A_1\frac{\partial T}{\partial x}+A_2 T = 0.$$

The straightforward numerical integration in time is proceeded until a steady-state is reached. For the calculations performed, a finite width B of the porous layer was chosen which had the same size as the layer thickness H_1. The side walls were assumed to be impermeable and adiabatic.

The region of the porous medium is covered by a grid



70	140	280	Ra_I	350	500	1000
$K/H_1^2=10^{-7}$ $\nu/\kappa_p=1.6\times10^{-2}$ $\kappa_L/\kappa_p=4$ $c^*=0.5$ $B/H_1=1$						

FIG. 2. Flow and temperature fields for steady-state convection.

of 17 × 17 mesh points. The numerical convergence of the finite-difference procedure was confirmed by employing grids of smaller mesh size on selected test problems. The numerical results are believed to be accurate to ±6% for integral quantities as, for example, the Nusselt number.

4. RESULTS AND DISCUSSIONS OF THE NUMERICAL CALCULATIONS

Figure 2 shows the steady-state convection pattern and the corresponding field of isotherms for different internal Rayleigh numbers. The following characteristic features can be observed.

When the Rayleigh numbers are increased, the width of the hot plume leaving the porous layer is reduced while the local velocities therein increase. On the other hand the range of cold liquid entering the heat generating layer is broadened. The maximum temperature in the porous layer is found in the hot plume close to the interface where a thermal boundary layer is formed as shown in the sketches of Fig. 2. Outside this thermal boundary layer the horizontally averaged temperature depends only weakly on the local height as shown in Fig. 3.

The heat transfer from the heat generating porous layer is described by a Nusselt number defined as the reciprocal of the averaged dimensionless temperature difference between the bottom of the porous and the top of the liquid layer

$$Nu = \frac{1}{\langle T_p \rangle} = \frac{QH_1^2}{\lambda_p \langle \theta_p - \theta_L \rangle}. \tag{14}$$

In Fig. 4 the numerical values of the Nusselt number are plotted vs the Rayleigh number. For comparison, experimental values of Nu obtained in refs. [9, 10] are also presented. In the range $100 \leqslant Ra_I \leqslant 3000$ the numerical results can be best fitted by a power law of the form

$$Nu = \left(\frac{Ra_I}{3.22}\right)^{0.533}. \tag{15}$$

The critical Rayleigh number defining the onset of convection has been adopted from the stability calculations of Turland [2].

The discrepancy between the numerical and experimental values of the Nusselt number reflects mainly the difference in the dimensionless permeability K/H_1^2. As outlined in Section 2, in the present work the permeability is limited to values $K/H_1^2 \leqslant 10^{-7}$ because of simplifying assumptions for the conditions at the interface. The experimental results of refs. [9, 10] were obtained from inductively heated steel-ball heaps saturated and covered by water. The permeabilities in these cases were in the ranges of $K/H_1^2 = 10^{-6}$–10^{-5} [10] and $K/H_1^2 = 4\times10^{-5}$–1.5×10^{-4} [9], respectively. Since the model of the present work does not apply for these values, no definite conclusion can be drawn regarding its validity. Nevertheless both the numerical and experimental results indicate that the Nusselt number increases with decreasing dimensionless permeabilities.

The fluid velocity distribution is closely related to the heat transfer in the porous layer. The maximum value of the velocity $v_{max} = (u^2+v^2)^{1/2}_{max}$ in the rising hot plume is

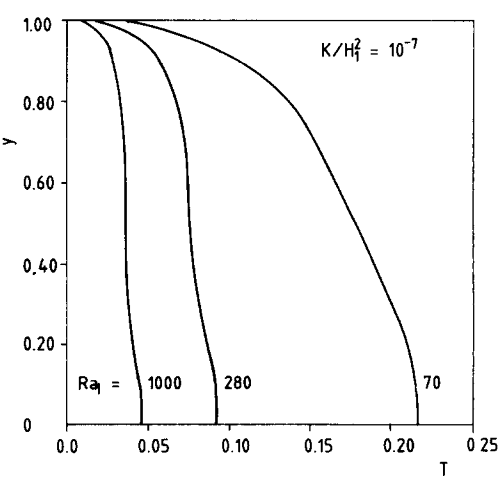
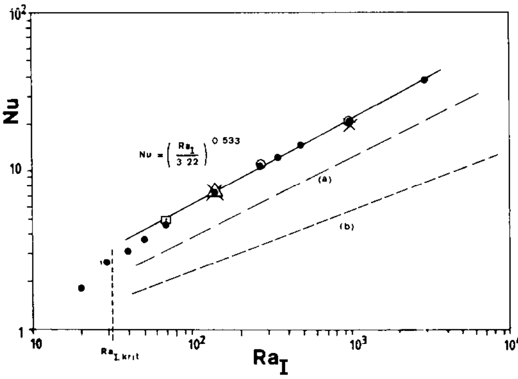


FIG. 3. Horizontally averaged temperature.



	B/H_1	K/H_1^2	Pr_p	c^*	κ_l/κ_p	Mesh points
●	1	10^{-7}	0 016	0.5	4	17x17
○	1	10^{-8}	0.016	0 5	4	17x17
×	1	10^{-7}	0.016	0.5	4	33x33
△	1	10^{-7}	3	1	1	17x17
□	1.5	10^{-7}	0.016	0.5	4	17x17

FIG. 4. Heat transfer from a porous layer. ●, ○, △, ×, and □ are numerical results for fuel particles in liquid sodium. (a) Experimental results, Barleon and Werle [10]. (b) Experimental results, Rhee *et al.* [9].

given in Fig. 5 as a function of the Rayleigh number. The numerical values can be correlated by the following power law

$$v_{\max} = \left(\frac{Ra_{\rm I}}{0.988}\right)^{0.534}. \tag{16}$$

For the safety analysis of nuclear reactors, the transient behaviour of the flow before reaching steady-state is also of some interest. From the solution of the time-dependent differential equations, equations (1)–(4), the horizontally averaged bottom temperature appears to reach its steady-state value in the form of a damped oscillation of period τ. Therefore, a dimensionless transient period of the convection within the porous bed can be defined as $\tau/(H_1^2/\kappa_p)$, which is also the Fourier number *Fo*. Except at the highest Rayleigh numbers, only a single period can be observed before

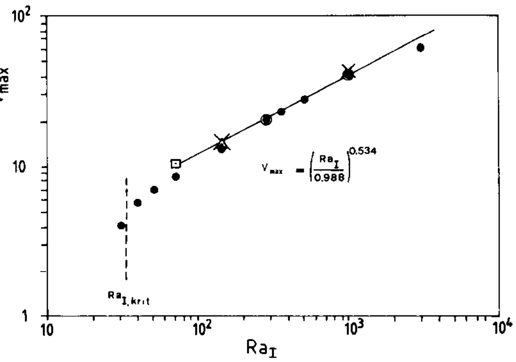


FIG. 5. Numerical results of the maximum velocity.

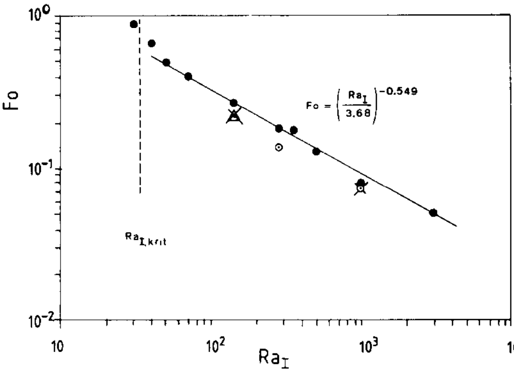


FIG. 6. Numerical results of the transient period.

the Nusselt number reaches 95% or more of its final value.

In Fig. 6 the Fourier number is plotted vs the Rayleigh number. The numerical data can be best fitted by a power law of the form

$$Fo = \left(\frac{Ra_{\rm I}}{3.68}\right)^{-0.549}. \tag{17}$$

It should be mentioned explicitly here that for all values of the Rayleigh numbers (even for values as high as a hundred times the critical value) for which time-dependent calculations were performed, steady-state convection was finally reached. This is rather surprising as it has been found, for convective flows in saturated porous layers heated from below and with impermeable upper boundaries, that oscillatory flow occurs at the order of ten times the critical Rayleigh number [11].

This different behaviour is believed to be due to the permeable interface.

As has been frequently done for the heat transfer from fluid layers heated from below and/or internally [13], the numerically derived Nusselt–Rayleigh number correlation is argued by analytical asymptotic considerations. This procedure will give further support to the arguments in Section 2 for simplifying the thermal boundary conditions at the interface.

5. ASYMPTOTIC ANALYSIS

For high Rayleigh numbers and small permeabilities, the main heat resistance in the double layer is concentrated in a thin thermal boundary layer at the interface (see Fig. 1).

For further considerations, let us concentrate our discussion on that portion of the boundary layer which is located in the porous layer. The average thickness of this portion of the boundary layer will be designated δ. According to Howard [12], the thickness of the Bénard-type thermal boundary layers is mainly determined by its stability. Following Cheung [13] and Hollands *et al.* [14], let us introduce as a limiting condition the following critical Rayleigh number for the thermal boundary layer

$$Ra_\delta = \frac{\alpha g K \delta c^* \langle \theta_p - \theta_1 \rangle}{\nu \kappa_p} = \text{const.}, \tag{18}$$

where the constant is of the order of less than 10^2 [2]. This postulation implies that the heat generated within the boundary layer can be neglected compared to the heat transferred through it.

From a heat balance at the interface we obtain

$$\left\langle -\frac{\partial T}{\partial y} \right\rangle + \langle v T_1 \rangle = 1, \quad \text{at } y = 1. \tag{19}$$

For the case of well developed boundary layers, the temperature gradient can be approximated as follows

$$\left\langle -\frac{\partial T}{\partial y} \right\rangle \approx \frac{\langle T_p - T_1 \rangle}{\delta / H_1}, \quad \text{at } y = 1. \tag{20}$$

For infinitely small permeabilities the convective heat transfer $\langle v T_1 \rangle$ across the interface can be neglected. If the boundary-layer thickness δ in equation (20) is eliminated by using the stability condition of equation (18) and the simplified heat balance of equation (19), we obtain the asymptotic relation

$$Nu \approx \left[\frac{Ra_I}{Ra_\delta}\right]^{0.5}. \tag{21}$$

Next, account will be taken of small but finite permeabilities which imply heat transfer by convection across the interface.

For assessing the convective heat transfer let us define a mean velocity v_m at the interface by the following relation

$$v_m = \frac{\langle v T_1 \rangle}{\langle T_1 \rangle}. \tag{22}$$

Using this definition, a heat balance for the temperature boundary layer yields

$$v_m \langle T_p \rangle - v_m \langle T_1 \rangle = \left\langle -\frac{\partial T}{\partial y}\bigg|_{y=1} \right\rangle, \tag{23}$$

where the heat generation within the boundary layer has been neglected compared to the total heat generation in the porous layer. Then by the approximation described by equation (20), it follows that

$$v_m \approx \frac{H_1}{\delta}. \tag{24}$$

This is equivalent to the relation

$$v_m \approx \frac{Ra_I}{Ra_\delta} \langle T_p - T_1 \rangle. \tag{25}$$

Thus we obtain for the convective heat transport across the interface

$$\langle v T_1 \rangle \approx \frac{Ra_I}{Ra_\delta} \langle T_p - T_1 \rangle \langle T_1 \rangle. \tag{26}$$

Inserting equation (22) and equations (20), (24), and (25) into equation (19) we obtain

$$\langle T_p - T_1 \rangle^2 \frac{Ra_I}{Ra_\delta} + \frac{Ra_I}{Ra_\delta} \langle T_p - T_1 \rangle \langle T_1 \rangle \approx 1,$$

and furthermore

$$\langle T_p - T_1 \rangle \approx -\frac{1}{2} \langle T_1 \rangle + \left[\frac{Ra_\delta}{Ra_I}\right]^{0.5} \left[1 + \frac{Ra_I}{Ra_\delta} \frac{\langle T_1 \rangle^2}{4}\right]^{0.5}, \tag{27}$$

where $\langle T_1 \rangle$ is now evaluated from equation (8) by using a simplification of the empirical heat transfer correlation, equation (9), in the form

$$Nu_L = 0.1 Ra_L^{1/3}. \tag{28}$$

Assuming that $\langle -\partial T / \partial y \rangle = 1$ in equation (8), we obtain

$$\langle T_1 \rangle = 5.6 \left(\frac{\lambda_p}{\lambda_L}\right)^{1/2} \left(\frac{K}{H_1^2}\right)^{1/4} \left(\frac{1}{Ra_I}\right)^{1/4}. \tag{29}$$

We mention here that in the heat balance, equation (8), heat transfer by convection is neglected because its contribution is small, of higher order in $(K/H_1^2)^{1/4}$, as can be seen by the combination of equations (26), (27), and (29).

Therefore, for the Nusselt number we obtain, to the lowest order in K/H_1^2, the following relation

$$Nu = \frac{1}{\langle T_p \rangle} \approx \left[\frac{Ra_I}{Ra_\delta}\right]^{0.5} \times \left[1 - \frac{5.6}{2} \frac{Ra_I^{1/4}}{Ra_\delta^{1/2}} \left(\frac{\lambda_p^{1/2}}{\lambda_L}\right) \left(\frac{K}{H_1^2}\right)^{1/4}\right]. \tag{30}$$

The constant value of Ra_δ can not be determined within the scope of our asymptotic considerations. However, from a comparison of the analytical and numerical results for the Nusselt number, Ra_δ can be determined to be

$$Ra_\delta \approx 2. \tag{31}$$

Our analytical considerations in this section lend support to our assumptions concerning the thermal conditions at the interface in Section 2, where we have limited the validity of both the numerical and the analytical model quantitatively to small values of the permeability K/H_1^2. More strictly speaking the condition

$$Ra_I^{1/4} \left(\frac{\lambda_p}{\lambda_L}\right)^{1/2} \left(\frac{K}{H^2}\right)^{1/4} \leqslant 0.1, \tag{32}$$

is required for reasonable accuracies in the application.

6. EXPERIMENTAL RESULTS

In order to reduce the discrepancy between experimental and numerical results, additional measurements of the heat transfer from a particle debris bed through a superposed fluid layer to an upper isothermal horizontal plate have been made. In the experiment a smaller dimensionless permeability K/H_1^2 was employed, because the asymptotic analysis indicated that the boundary conditions used in the computation were only valid for small permeabilities.

The experimental apparatus consisted of an insulated vertical glass tube of 8 cm I.D., in which steel or bronze spheres of 1–3 mm diameter could be piled up to a height of 20 cm. The debris bed was saturated and superposed by water. A horizontal ceramic cooling plate was inserted into the cylinder forming an isothermal upper boundary. The height H_2 of the plate above the interface could vary between 0.5 and 10 cm. The debris were heated inductively with a copper coil surrounding the glass jar, temperatures were measured with various thermocouples. For a more detailed description of the apparatus we refer to ref. [15].

For technical reasons only a small aspect ratio of the porous bed could be realized. Two-dimensional calculations were performed for this particular case choosing an aspect ratio $B/H_1 = 0.354$. This implies the assumption, that an analogy exists for the flow conditions in containers of cylindrical and square cross-sections and the same height. For this case a higher critical Rayleigh number and a different correlation $Nu(Ra_I)$ was obtained for small but supercritical Rayleigh numbers up to $Ra_I \approx 15\,000$. For $70 \leqslant Ra_I \leqslant 15\,000$ the following correlation was derived

$$Nu = \left(\frac{Ra_I}{27.4}\right)^{0.707}. \tag{33}$$

For higher Rayleigh numbers a correlation in the form of equation (15) was obtained again.

Numerical predictions and experimental results are compared in Fig. 7. At subcritical Rayleigh numbers the experimental data for the Nusselt number increase slightly with Ra_I. This effect is attributed to the increasing convection within the fluid layer. According to equation (29) the Nusselt number can be approximated in this range by

$$Nu = 2\frac{11.6}{Ra_I^{1/4}}\left(\frac{K}{H_1^2}\right)^{1/4}\left(\frac{\lambda_P}{\lambda_L}\right)^{1/2}. \tag{34}$$

Data obtained with fine bronze particles for this range of Rayleigh numbers prove, that the turbulent convection of the fluid layer does not induce any significant motion within the porous bed.

At high Rayleigh numbers the experimental data follows the calculated slope of the Nusselt number, but even for the smallest permeabilities the experimental values of the Nusselt numbers are lower by 30% compared to the ones predicted by the numerical calculations. However, the tendency of increasing the Nusselt numbers when decreasing the permeabilities is confirmed again. In general the comparison between experimental and theoretical data indicates that the conditions at the interface assumed in the analysis were not precise enough to obtain better agreement.

7. CONCLUSIONS

Heat removal by natural convection from a particle bed saturated and superposed by fluid has been investigated by theory and experiments. The investigations were limited to the special case when convection in the superposed fluid layer is turbulent and when the permeability is sufficiently small. With these assumptions the conditions at the interface could be simplified, and convection within the porous layer could be calculated separately. The time dependent 2-D

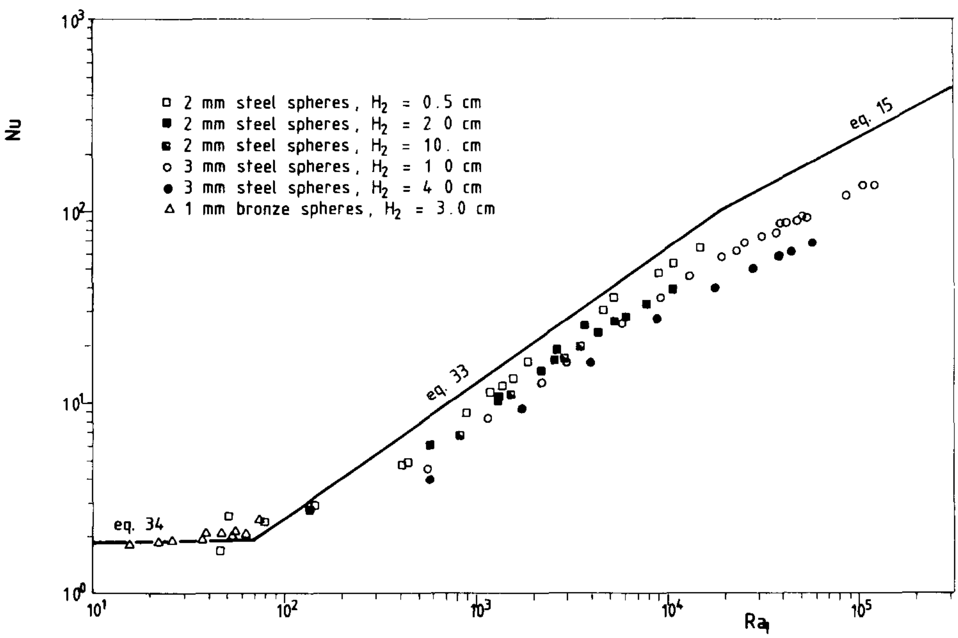


FIG. 7. Experimental results of heat transfer from the porous bed taken from ref. [15] and comparison with numerical predictions.

problem was solved numerically. The Nusselt number and the Fourier number as well as the maximum velocity are correlated by simple power laws of the internal Rayleigh number. Moreover, a 1-D asymptotic model has been derived which confirms the dependence of the Nusselt number at high Rayleigh numbers obtained by numerical methods. The asymptotic model correlates the Nusselt number linearly with other parameters, which include the influence of the superposed fluid layer on the overall heat transfer. The model also quantifies the range of validity of the employed interface conditions.

Additional measurements have been performed to validate the numerical results. The measurements show that in case a smaller permeability of the bed is used, the experimental data approach the predictions of the calculations. Still existing discrepancies between experimental and theoretical data are attributed to the fact, that the conditions at the interface and the turbulent motion within the fluid layer have not been modelled properly enough. A more detailed theory or further experiments will be necessary to refine this model.

Neither in the numerical computations nor in any of the experiments is an oscillatory instability at high Rayleigh numbers or a subcritical convection within the porous bed observed.

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APPENDIX

According to Nield [3], at the interface the pressures within the porous medium and the fluid layer differ by the normal stress, which in dimensionless form gives

$$p_p = p_L - 2\frac{K}{H_1^2}\frac{\partial v_L}{\partial y}; \quad y = 1. \tag{A1}$$

Differentiating this equation with respect to x and using the continuity equation, we obtain

$$\frac{\partial p_p}{\partial x} = \frac{\partial p_L}{\partial x} + 2\frac{K}{H_1^2}\frac{\partial^2 u_L}{\partial x^2}; \quad y = 1. \tag{A2}$$

The pressure gradient in the liquid layer can be determined by employing the momentum equation. In dimensionless form, we obtain

$$\frac{\partial p_L}{\partial x} = -\frac{K}{H_1^2}\frac{1}{Pr}\frac{\lambda_p}{\lambda_L}\left(u_L\frac{\partial u_L}{\partial x} + v_L\frac{\partial u_L}{\partial y}\right) + \frac{K}{H_1^2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)u_L. \tag{A3}$$

Combining equations (A3) and (A2) we have

$$\frac{\partial p_p}{\partial x} = -\frac{K}{H_1^2}\frac{1}{Pr}\frac{\lambda_p}{\lambda_L}\left(u_L\frac{\partial u_L}{\partial x} + v_L\frac{\partial u_L}{\partial y}\right) + \frac{K}{H_1^2}\left(3\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)u_L; \quad y = 1. \tag{A4}$$

Because of continuity, the velocity vectors (u_L, v_L) and (u_p, v_p) must be of the same order close to the interface. The pressure gradients in the x-direction in the porous layer [see equation (2) and at the interface equation (A4)] differ therefore by an order of

$$O\left(\frac{K}{H_1^2}\frac{1}{Pr}\frac{\lambda_p}{\lambda_L}\right). \tag{A5}$$

If we restrict ourselves to porous media of small permeabilities K/H_1^2, $\partial p_p/\partial x$ can be neglected to this order of magnitude at the interface. From this it follows

$$p = \text{const.} + O\left(\frac{K}{H_1^2}\right); \quad y = 1. \tag{A6}$$

CONVECTION NATURELLE DANS DES COUCHES POREUSES SATUREES AVEC SOURCES THERMIQUES INTERNES

Résumé—Le problème de la convection bidimensionnelle dans une couche poreuse saturée et chauffée intérieurement est étudié numériquement par les différences finies. La couche poreuse est surmontée par une couche fluide et posée sur une base adiabatique et imperméable. Se limitant aux faibles perméabilités, on suppose une pression constante à l'interface entre le milieu poreux et la couche liquide supérieure et la température y est fixée par une relation empirique de flux thermique pour une convection pleinement turbulente dans la couche liquide. Partant d'une distribution initiale de température, on obtient la convection permanente pour des nombres de Rayleigh allant jusqu'à cent fois la valeur critique. Les nombres de Nusselt et de Fourier sont présentés sous forme de lois puissances du nombre de Rayleigh.

NATÜRLICHE KONVEKTION IN GESÄTTIGTEN PORÖSEN SCHICHTEN MIT INNEREN WÄRMEQUELLEN

Zusammenfassung—Mit einem Differenzen-Verfahren wurde zweidimensionale Konvektion in einer porösen Schicht mit inneren Wärmequellen untersucht. Die poröse Schicht ist von einer Flüssigkeitsschicht überlagert und auf einer undurchlässigen und adiabaten Unterlage angeordnet. Unter Einschränkung auf den Fall geringer Durchlässigkeiten wird ein konstanter Druck an der Grenzfläche zwischen gesättigtem porösen Medium und der darüberliegenden Flüssigkeitsschicht angenommen; die Temperatur an dieser Stelle wird durch eine empirische Wärmestromgleichung für voll-turbulente Konvektionsströmung innerhalb der Flüssigkeitsschicht festgelegt. Ausgehend von einer Anfangstemperaturverteilung, wurde stationäre Konvektion für gegebene Rayleigh-Zahlen bis zum Hundertfachen des kritischen Wertes gefunden. Nußelt- und Fourier-Zahlen werden in Form von Potenzansätzen der Rayleigh-Zahl angegeben.

ЕСТЕСТВЕННАЯ КОНВЕКЦИЯ В НАСЫЩЕННЫХ ПОРИСТЫХ СЛОЯХ С ВНУТРЕННИМИ ИСТОЧНИКАМИ ТЕПЛА

Аннотация—Методом конечных разностей численно исследована задача двухмерной конвекции в нагреваемом изнутри насыщенном пористом слое. Над пористым слоем на непроницаемом адиабатическом покрытии расположен слой жидкости. При небольшой проницаемости предполагается постоянство давления на границе раздела между насыщенной пористой средой и слоем жидкости. Температура определяется величиной теплового потока для полностью развитой турбулентной конвекции в слое жидкости. При исследованных начальных распределениях температуры конвекция для заданных чисел Рэлея оставалась стационарной вплоть до значений, в сто раз превышающих критическое. Числа Нуссельта и Фурье представлены в виде степенной зависимости от числа Рэлея.